Alternative Accelerated CC Math 6/7 – UNIT 4
Understanding Expressions and Equations

Critical Area: Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

Rationale: The Expression and Equations unit begins with standards 6.EE.1 and 6.EE.2. By starting with these two standards, students are learning basic skills needed to read and write expressions, which are perquisites for manipulating equations. It is recommended Standards 6.EE.2a and 6.EE.2b be taught concurrently. By blending both standards, students will use mathematical terms while learning how to write expressions. It is also suggested to use the geometry standards (6.G.1, 6.G.2 and 6.G.4) to teach standard 6.EE.2c. 6.EE.2c, which requires students to solve real world problems using area, surface area, and volume. Students are learning the connections between the standards, not learning each standard in isolation. In the next standard in the sequence 6.EE.3, students apply and further develop their conceptual understanding of arithmetic to algebraic expressions from third grade. 6.EE.3 is followed by 7.EE.1 and 7.EE.2 because this is the application of 6.EE.3. Students are applying their knowledge of properties to generate equivalent expressions. After students have completed 7.EE.1 and 7.EE.2, the idea of equivalent expression returns with standards 7.EE.4. This standard (7.EE.4) will help connect the idea of equivalent expressions to equations. In standards 6.EE.5 through 6.EE.9 students develop their understanding of expressions, equations, and the relationship between independent and dependent variables, which are perquisites for Standards 7.EE.3 and 7.EE.4. In Standards 7.EE.3 and 7.EE.4 students apply the knowledge of expression and equations to real world applications.

CLUSTERS	COMMON CORE STATE STANDARDS
Apply and extend previous understandings of arithmetic to algebraic expressions	6.EE.1. Write and evaluate numerical expressions involving whole-number exponents.
Apply and extend previous understandings of arithmetic to algebraic expressions. **(Blended 2a and 2b)**	6.EE.2. Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.* b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including

Supporting Standards for 6.EE.2c: Solve real-world and mathematical problems involving area, surface area, and volume. **(Teach standards 6.EE.2c using the geometry standards)**	those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6 s^2$ to find the volume and surface area of a cube with sides of length s = 1/2.* 6.G.1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. 6.G.2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l w h$ and $V = b h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. 6.G.4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.
Apply and extend previous understandings of arithmetic to algebraic expressions	6.EE.3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*
Use properties of operations to generate equivalent expressions	7.EE.1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients 7.EE.2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, a + 0.05a = 1.05a means that *"increase by 5%"* is the same as *"multiply by 1.05."*
Apply and extend previous understandings of arithmetic to algebraic expressions	6.EE.4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which*

	number y stands for.
Reason about and solve one-variable equations and inequalities.	6.EE.5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. 6.EE.6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. 6.EE.7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. 6.EE.8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x <$ c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
Represent and analyze quantitative relationships between dependent and independent variables.	6.EE.9.Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.
Solve real-life and mathematical problems using numerical and algebraic expressions and equations	7.EE.3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2* *inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

	7.EE.4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p,q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Writean inequality for the number of sales you need to make, and describe the solutions.*
STANDARDS IN CCSS MATH 6	PREREQUISITE COMMON CORE STATE STANDARDS
6.EE.1. Write and evaluate numerical expressions involving whole-number exponents.	**5**.nbt.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. 4.oa.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
6.EE.2. Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.* b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view	6.EE.1

one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6 s^2$ to find the volume and surface area of a cube with sides of length s = 1/2.*	
6.G.1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	4.MD.3 Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.* *5.NF.4b* Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
6.G.2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	5.MD.5a Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. 5.MD.5b Apply the formulas $V = l\ x\ w\ x\ h$ *and* $V = b\ x\ h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of

	solving real-world and mathematical problems.
6.G.4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	6.G.1 5.NF.4 4.MD.3
6.EE.3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*	6.EE.1 3.oa.5 Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56 (Distributive property)*
7.EE.1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients 7.EE.2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, a + 0.05a = 1.05a means that *"increase by 5%"* is the same as *"multiply by 1.05."*	6.EE.3 3.OA.5 Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56 (Distributive property)*
6.EE.4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*	6.EE.1 5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*
6.EE.5. Understand solving an equation or inequality as a	6.EE.4

process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	6.EE.2c
6.EE.6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	6.EE.2
6.EE.7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.	6.EE.6 6.EE.5
6.EE.8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x <$ c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	6.EE.7
6.EE.9.Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.	6.EE.5 6.EE.6
7.EE.3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools	6.EE.8

strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2* *inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*	
7.EE.4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.	
a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*	6.EE.7 6.EE.9
b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p,q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*	6.EE.8

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
The middle grades standards in Expression and Equations build a ramp from arithmetic in elementary school to more sophisticated work with algebraic expression in high school. As the complexity of expressions increase, students will see them as being built out of basic operations with products and factors. During the middle grades, algebraic expressions are simply numbers in which one or more letters are used to stand for a number which is either unspecified or unknown. Later in high school, students learn to use the properties of operations to write expressions in different but equivalent forms. At some point they see equivalent expressions, particularly polynomial and rational expressions, as naming some underlying thing. The repertoire of functions that is acquired during high school allows students to create more complex equations, including equations arising from linear and quadratic functions, and simple rational and exponential functions; students will no longer be limited largely to linear equations in modeling relationships between quantities with equations in two variables; and students start to work with inequalities and systems of equations.	How can you apply the properties of operations to generate equivalent expressions? Which values from a specified set, if any, make an equation or inequality true? In what ways can you reason and solve one-variable equations and inequalities? How do expressions and equations apply to real life situations? How might an inequality describe a real-life problem? Can you show how inequalities can have infinitely many solutions? In what ways can you show the relationship between dependent and independent variables? How can I apply the order of operations and the fundamentals of algebra to solve problems? How can I justify that multiple representations in the context of a problem are equivalent expressions? How do I assess the reasonableness of my answer? How will I use the properties of equality to explain the order of the steps in solving equations and inequalities? How do I interpret the solutions for equations and inequalities in the context of the problem?	Expression Equation Inequality Formulas Variables Equivalent Coefficient constants Terms Identity properties of addition and multiplication Distributive property Associative property Commutative property Identity properties of addition and multiplication Rational numbers Dependent variable Independent variable Solution Solution set Properties Operations Factor Expand Linear Rational Coefficient Inequality Algebraic Arithmetic Context Per Square Root Cube Root Perfect Square Perfect Cube

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
Sample Performance Task from Smarter Balanced 1. Design a Garden: Grade 6 PT Claim 4 -Standards: 6.EE.9, 6.G.1, 6.G.2, 5.MD.3, 5.MD.5, 5.OA.2, 4.MD. -Math Practice Standards: 1, 3, 4, and 5 2. School Fundraisers: Grade 7 PT Claim 4 -Standards: 7.EE.1, 7.EE.2, 7.EE.3, 7.EE.4, 7.RP.1, 7.RP.2, 7.RP.3, 7.SP.1, 7.SP.4, 6.EE.4, 6.EE.5, 6.EE.6, 6.RP.1, 6.RP.3, 6.SP.1, 6.SP.5 -Math Practice Standards: 1, 2, 3, 4, 6, 7, 8 **Mathematics Assessment Project** *Problem Solving* 1. Investigating: Consecutive Sums: Grade 6 - Standard: 6.EE.3 http://map.mathshell.org/materials/lessons.php?taskid=598&subpage=problem 2. Modeling: Car Skid Marks: Grade 6 -Standard; 6.EE.9 http://map.mathshell.org/materials/lessons.php?taskid=588&subpage=problem *Concept Development* 1. Laws of Arithmetic: Grade 6 - Standards: 6.EE.3 and 6.G. http://map.mathshell.org/materials/lessons.php?taskid=484&subpage=concept	Teachers are strongly encouraged to use algebra tiles or "Hands On Equations"© as students are developing a connection from concrete mathematical representations to abstract notions of variables. Spreadsheets are a powerful tool to help students understand the concept of variable because you can use formulas that are dependent on the values in a cell and then change the value in the cell. It is very easy for students to see how changing the value of the variable affects the value of the cell with the formula. Whole class response tools (such as index cards, white boards, and electronic response devices) allow teachers to check for understanding before moving forward with new material.	SBAC - http://www.smarterbalanced.org/ PARCC - http://parcconline.org/samples/mathematics/grade-6-slider-ruler

2. Interpreting Equations: Grade 6 -Standard: 6.EE http://map.mathshell.org/materials/lessons.php?taskid=580&subpage=concept 3. Using Positive and Negative Numbers in Context: Grade 7-Standards: 7.NS.1 and 7.EE.1 http://map.mathshell.org/materials/lessons.php?taskid=453&subpage=concept 4. Steps to Solving Equations: Grade -Standards: 7.EE1 and 7.EE.4 http://map.mathshell.org/materials/lessons.php?taskid=431&subpage=concept **Illustrative Mathematics** 1. Anna in DC https://www.illustrativemathematics.org/illustrations/997 2. Guess My Number https://www.illustrativemathematics.org/illustrations/712 3. Gotham City Taxis https://www.illustrativemathematics.org/illustrations/884 4. Book Store Account https://www.illustrativemathematics.org/illustrations/1475 5. Stained Glass https://www.illustrativemathematics.org/illustrations/1513 6. Sierpinski's Carpet https://www.illustrativemathematics.org/illustrations/1523 7. Pennies to Heaven https://www.illustrativemathematics.org/illustrations		

/1291 8. Triangular Tables https://www.illustrativemathematics.org/illustrations/494 9. Fruit Salad https://www.illustrativemathematics.org/illustrations/1032 10. Chocolate Bar Sales https://www.illustrativemathematics.org/illustrations/806		

LANGUAGE GOALS

- Students will describe their understanding of properties of operations to generate equivalent fraction, using the words distributive, associative, commutative, and identity properties.
- Students will accurately read equivalent expressions aloud fluently, without hesitating.
- Students will ask and answer why values from a specified set, if any, make an equation or inequality true using equations and expressions.
- Students will write an opinion to show how inequalities can have infinitely many solutions. The key to determining that the inequalities have _____. This possible because ____. I believe this because ____.
- Students will distinguish between dependent and independent variables and describe the relationship between them using sentence starters such as:
  I think _____ is the dependent variable because _____.
  The relationship between_____ and _____ is _____.
- Students will explain how to use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity.

PERFORMANCE TASKS

Mathematics Assessment Project
Laws of Arithmetic: http://map.mathshell.org/materials/download.php?fileid=1358

Optimizing: Security Cameras: http://map.mathshell.org/materials/download.php?fileid=1354

Mean, Median, Mode, and Range: http://map.mathshell.org/materials/download.php?fileid=1360

DIFFERENTIATION 🕮

FRONT LOADING	ACCELERATION	INTERVENTION
Students apply and extend understandings using numerical expressions. They use whole number exponents to express powers of 10; using letters to represent an unknown quantity. They also move from viewing expressions as actions describing a calculation to viewing then as objects in their own right (concrete to abstract). In grades k-5 students have been using properties of operations to write expression in different ways. These experiences with properties help students prepare for work with algebraic expressions. For example students in grades k-5 have been writing numerical expressions and simple equations involving one operation with a variable.	Encourage students to individualize their learning by providing them with the tools to further investigate concepts that will be developed further in other grade levels. For example, Although the process of reasoning will eventually lead to standard methods for solving equations, students should study examples where looking for structure pays off, such as in $4x + 3x = 3x + 20$, where they can see that $4x$ must be 20 to make the two sides equal. This understanding can be reinforced by comparing arithmetic and algebraic solutions to simple word problems. For example, how many 44-cent stamps can you buy with $11? Students are accustomed to solving such problems by division; now they see the parallel with representing the problem algebraically as $0.44n = 11$, from which they use the same reasoning as in the numerical solution to conclude that $n = 11/0.44$. Interdisciplinary connections can be made to Social Studies units were math enables history to be explained in more concrete ways. For example population growth rates.	Transitioning from concrete to abstract is important and needs to be a part of intervention. A clear connection between symbolic representation and expression is key. • Small teacher to student ratio discussion. For example, describing the relationship between distance and time for a person starting 5 miles from home and walking away at 5 miles per hour. With these types of discussions students begin to develop an understanding of variables. • Emphasize think-pair-share. • Students can use tabular and graphical representations to develop an appreciation of varying quantities. • Make connections to real life

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7. California Department of Education. (2013). Draft Mathematics Framework Chapters. Retrieved from http://www.cde.ca.gov/be/cc/cd/draftmathfwchapters.asp.
8. National Council of Teachers of Mathematics (NCTM) Illuminations. (2013). Retrieved from http://illuminations.nctm.org/Weblinks.aspx.
9. The University of Arizona. (2011-12). Progressions Documents for the Common Core Math Standards. Retrieved from http://ime.math.arizona.edu/progressions.