

Statistics and Probability – UNIT 5
Making Inferences and Justifying Conclusions

Introduction: Instructional time will be spent on making inferences and justifying conclusions based on simulations. Students will simulate sampling distributions to estimate a population proportion or mean and develop the concept of margin of error. Students will use these simulated sampling distributions to make decisions whether differences between parameters are significant.

| CLUSTER | COMMON CORE STATE STANDARDS |
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| <p>Make inferences and justify conclusions from statistical studies.</p> | <p><u>HSS.IC.B.4</u> Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.</p> <p><u>HSS.IC.B.5</u> Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.</p> |
| MATHEMATICAL PRACTICES | LEARNING PROGRESSIONS |
| <p>1. Make sense of problems and persevere in solving them.</p> <p>2. Reason abstractly and quantitatively.</p> <p>3. Construct viable arguments and critique the reasoning of others.</p> <p>4. Model with mathematics.</p> <p>5. Use appropriate tools strategically.</p> <p>6. Attend to precision.</p> <p>7. Look for and make use of structure.</p> <p>8. Look for and express regularity in repeated reasoning.</p> | <p>Statistics and Probability Progression http://commoncoretools.me/wp-content/uploads/2012/06/ccss_progression_sp_hs_2012_04_21_bis.pdf</p> |

| ENDURING UNDERSTANDINGS | ESSENTIAL QUESTIONS | KEY VOCABULARY |
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| <ul style="list-style-type: none"> Students understand simulation of random behavior and how to simulate a sampling distribution. Students understand the Central Limit Theorem. Students understand confidence intervals and critical values. Students construct and interpret a confidence interval for a population proportion or a population mean. Students learn how to calculate and interpret the margin of error. Students learn to estimate a population proportion and mean. Students learn to write hypotheses and make decisions based on the results of a simulation. Students learn to interpret Type I and Type II errors in context, and give the consequences of each. | <ul style="list-style-type: none"> What is the purpose of a confidence interval? How does the Central Limit Theorem apply to sampling distributions? How do you construct and interpret a confidence interval? What are the factors that affect the margin of error of a confidence interval? How do you estimate a population proportion or a population mean? What is the difference between a null and alternate hypothesis? How do you interpret a simulation in order to make a decision about a hypothesis? What is statistical significance? After making a conclusion, what types of error could be made? How do you describe Type I and Type II errors in context? | Alternate hypothesis Central Limit Theorem Confidence interval Confidence level Critical values Hypothesis testing Margin of error Null hypothesis P-value Sample proportion Sample test statistics Sampling distributions Standard error of the mean Statistical significance Type I error Type II error |

| RESOURCES | INSTRUCTIONAL STRATEGIES | ASSESSMENT |
|---|---|--|
| <ul style="list-style-type: none"> Confidence interval applet: http://bcs.whfreeman.com/tps5e/ StatKey (lock5stat.com/statkey): Create sampling distributions to analyze confidence intervals and p-values more efficiently <i>Statistical Reasoning in Sports</i>, Tabor, J & Franklin, C. <i>Workshop Statistics: Discovery with Data</i>, 4th ed., Rossman, A & Chance, B. <i>Statistics Through Applications</i>, 2nd ed., Starnes, Yates, & Moore. What Is Happening in This Class? (WIHC): Student questionnaire http://www.chemeng.ntua.gr/courses/ped2/files/WIHC.pdf | <ul style="list-style-type: none"> Incorporate various activities into this unit. Since the students will be making inferences only through a simulation-based approach, be sure to allow the students to work with data in a hands-on manner as often as possible. For example, you may have the students use coins, dice, cards, or spinners to create an approximate sampling distribution. Then, you may use technology (Fathom, Statkey, etc.) to re-create a sampling distribution with a large number of trials. Students will be more likely to understand the idea of a p-value and a margin of error after participating in numerous hands-on simulations in class. When writing hypotheses, emphasize that the null hypothesis is usually a statement of “no | <p>Formative Assessment SBAC - http://www.smarterbalanced.org/</p> <hr/> <p>LAUSD Periodic Assessment District assessments can be accessed through: http://achieve.lausd.net/math http://achieve.lausd.net/ccss</p> <p>Use your Single Sign On to access the Interim Assessments</p> <hr/> <p>California will be administering the SMARTER Balance Assessment as the end of course for grades 3-8 and 11. The 11th grade assessment will include</p> |

| RESOURCES | INSTRUCTIONAL STRATEGIES | ASSESSMENT |
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| <ul style="list-style-type: none"> • <i>Against All Odds</i>: https://www.learner.org/resources/series65.html • Rossman Chance Applet collection (http://www.rossmanchance.com/applets/index.html): Reeses Pieces Applet for sampling distributions • AP Stats Monkey: This site includes a wonderful collection of resources written by teachers and collected by Jason Molesky. http://apstatsmonkey.com/StatsMonkey/Statsmonkey.html | <p>effect” or “no difference.” Also, emphasize that the hypotheses are always stated in terms of a parameter, not in terms of the statistic that is collected.</p> <ul style="list-style-type: none"> • A single estimate of a population parameter is called a “point estimate” since it represents a single point on the number line. • Help students understand that the margin of error in a confidence interval accounts for variability due only to random selection or random assignment; it does not compensate for any bias in the data collection process. • Help students make connections between the formulas in this unit and previously learned formulas (mean, standard deviation, z-score, etc.). • Provide students with sentence frames to help them interpret confidence intervals and confidence level. • When students write the conclusion to a simulation question, emphasize that the answer is always an approximation. | <p>items from all High School Common Core strands, including Statistics and Probability. For examples, visit the SMARTER Balance Assessment at: http://www.smarterbalanced.org/</p> |

LANGUAGE GOALS for **low achieving, high achieving, students with disabilities and English Language Learners**

- Students will discuss and interpret confidence intervals for a population proportion or a population mean.

Example: The confidence interval for _____ is (____, ____). Interpret this interval in context.

- Students will write all inference conclusions using the language of the original question of interest.

Example Stem: Since the probability of the observed result or more extreme occurring is **high/low**, we **have/do not have** convincing evidence to conclude that _____ (alternative hypothesis in context) is true.

- Students will use data from a randomized experiment to compare, in writing, two treatments.

Example: Treatment A was more effective than treatment B because...

- Students will read closely to determine the appropriate inference procedure for a given scenario.

Example: I would choose _____ (inference procedure) because...

PERFORMANCE TASK

1. A survey was conducted involving 250 out of 125,000 families living in a city. The average amount of income tax paid per family in the sample was \$3540 with a standard deviation of \$1150. Construct and interpret a 99% confidence interval for the total taxes paid by all the families in the city.
2. During an angiogram, heart problems can be examined via a small tube (a catheter) threaded into the heart from a vein in the patient's leg. It is important that the company that manufactures the catheter maintain a diameter of 2.00 millimeters (mm). Each day, quality control personnel make several measurements to ensure that the diameter has not changed. If they discover a problem that causes the diameter to be different than 2.00 mm, they will stop the manufacturing process until it is corrected. Suppose on one specific day, quality control personnel take a random sample of 35 measurements and find the mean to be 2.009 mm and the standard deviation to be 0.021 mm.
 - a) Find the standard error of the sample mean for samples of size 35. Interpret this value in context.
 - b) Do these data give convincing evidence that the company should stop the manufacturing process? Use a significance test with $\alpha = 0.05$ to find out.
 - c) Calculate a 95% confidence interval for μ . Does your interval support your decision from part (b)?
3. Provide students with a published survey or have them collect data ($n \geq 30$). Analyze the data to estimate a population mean and develop a margin of error.

| DIFFERENTIATION  | | |
|--|--|--|
| UDL/ FRONT LOADING | ACCELERATION | INTERVENTION |
| <p>Statistics and Probability:</p> <ul style="list-style-type: none"> As an opening activity to help demonstrate the concept of sampling distributions is to take a large population of pennies and have the students take SRSs of size 1, 5, and 20 and find the sample mean year for each sample. Have the students plot the distributions on three different chart papers. This should also help demonstrate the Central Limit Theorem. Review the differences between a sample and a population, and a statistic and a parameter. | <p>Acceleration for high achieving students:</p> <ul style="list-style-type: none"> Students further their study in inference by learning specific hypothesis tests (z-test, t-test) and how to utilize a graphing calculator to run each test. Discuss inferences for categorical data (chi-square) and for the slope of a regression line. Discuss different transformations to achieve linearity (power, log, etc.). | <ul style="list-style-type: none"> Remind students that the sampling distribution of a statistic tells us the possible values of the statistic and how likely they are to occur. Remind students that parameters are fixed, and that there is nothing “special” about one confidence interval (i.e. the one sample that was taken may not even capture the parameter). Emphasize that, when conducting a hypothesis tests, an outcome that would rarely occur if a claim were true is good evidence that the claim is false. This may help students better understand the idea of a significance level. |