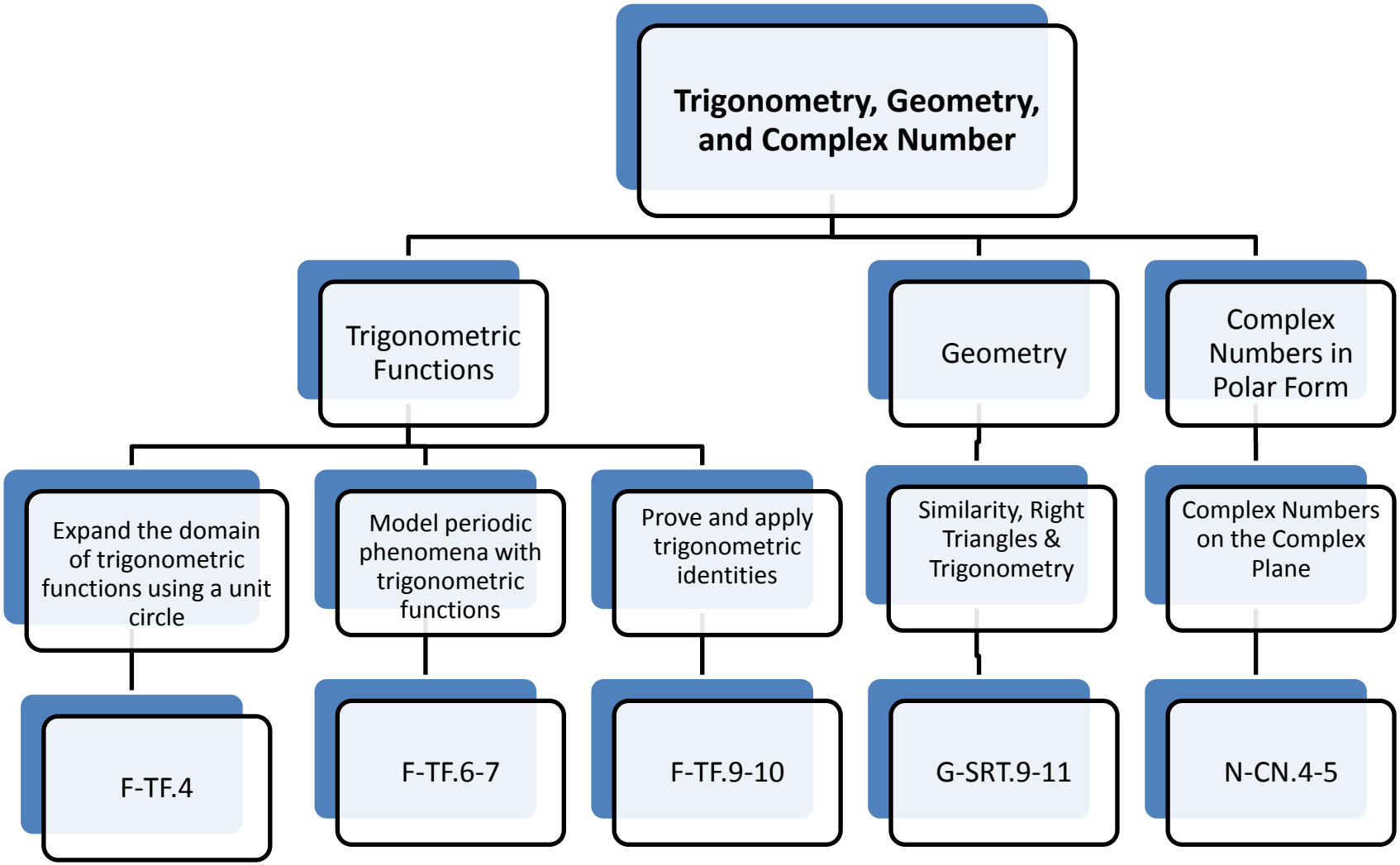
Precalculus
Unit 3
Trigonometry



Pre-Calculus – UNIT 3

Trigonometry

Critical Area: Students expand their understanding of the trigonometric functions by connecting properties of the functions to the unit circle, e.g., understanding that since that traveling 2π radians around the unit circle returns one to the same point on the circle, this must be reflected in the graphs of sine and cosine. Students extend their knowledge of finding inverses to doing so for trigonometric functions, and use them in a wide range of application problems. Students derive the addition and subtraction formulas for sine, cosine and tangent, as well as the half angle and double angle identities for sine and cosine, and make connections between among these. The relationships of general triangles using appropriate auxiliary lines result in the Laws of Sines and Cosines in general cases, and they connect the relationships described to the geometry of vectors. Students investigate the geometry of the complex numbers more fully and connect it to operations with complex numbers. In addition, students develop the notion of a vector and connect operations with vectors and matrices to transformations of the plane.

CLUSTERS	COMMON CORE STATE STANDARDS
	Functions – Trigonometric Functions
Expand the domain of trigonometric functions using a unit circle.	**F-TF.4**. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
Model periodic phenomena with trigonometric functions	**F-TF.6.** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. **F-TF.7.** Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
Prove and apply trigonometric identities	**F-TF.9.** Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. **F-TF.10.** Prove the half angle and double angle identities for sine and cosine and use them to solve problems.
Similarity, Right Triangles & Trigonometry	**Geometry – Similarity, Right Triangles, and Trigonometry** **G-SRT.9.** Derive the formula $A = 1/2\ ab\ \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. **G-SRT.10.** (+) Prove the Laws of Sines and Cosines and use them to solve problems. **G-SRT.11.** (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown

CLUSTERS	COMMON CORE STATE STANDARDS
	measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
Complex Numbers on the Complex Plane *[Revisit]*	**Number and Quantity – Complex Number** **N-CN.4.** (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. **N-CN. 5.** (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\ i)^3 = 8$ *because* $(-1 + \sqrt{3}\ i)$ *has modulus* 2 *and argument* 120°.
MATHEMATICAL PRACTICES	PROGRESSION
1. Make sense of problems and persevere in solving them. **2. Reason abstractly and quantitatively.** **3. Construct viable arguments and critique the reasoning of others.** **4. Model with mathematics.** **5. Use appropriate tools strategically.** **6. Attend to precision.** **7. Look for and make use of structure.** 8. **Look for and express regularity in repeated reasoning.**	

★Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.

(+) Indicates additional mathematics to prepare students for advanced courses.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• Trigonometric relationships and functions could be used to model real-world phenomenon. • Indirect measurements of lengths and angles can be used to solve a variety of problems. • The characteristics of circular functions and their representations are useful in solving real-world problems. • The relationship between the graph of a complex number and their operations and the conjugation of complex numbers on the complex plane can be understood.	1) How can the graphs of the sine, cosine, tangent functions and their inverses be compared? 2) How can you use the addition and subtraction formulas for sine, cosine, and tangent to solve problems? 3) How can you find the inverse of a trigonometric function? 4) How can you solve trigonometric equations using inverse functions? 5) How can technology be used to evaluate solutions of	• amplitude • asymptote • complex number • cosecant (csc) • cosine (cos) • cotangent (cot) • coterminal angles • even function • inverse • midline

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• The proof of addition and subtraction of identities are derived from the unit circle. • Domain must be limited to finding the inverse of a trigonometric function. Inverse functions must be used to find solutions in some modeling problems.	trigonometric functions? 6) How can you graph a complex number in rectangular and polar form? 7) What is the relationship between rectangular and polar form of a complex number? 8) What is the importance of knowing the conjugate of a complex number? In terms of their respective equations, what is the difference between a circle and an ellipse?	• odd function • period • periodic functions • phase shift • polar form • quadrantal angles • rectangular form • secant (sec) • sine (sin) tangent (tan)

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
Illustrative Mathematics • Axonometry: N-CN.4, N-CN.5 http://illuminations.nctm.org/Lesson.aspx?id=4228 • Graphs from the Unit Circle: F-TF.4 http://illuminations.nctm.org/Lesson.aspx?id=2870 • http://illuminations.nctm.org/Search.aspx?view=search&kw=identities&gr=9-12 • Cutting Conics: G-GPE.3 http://illuminations.nctm.org/Lesson.aspx?id=2907 • Shrinking Candles: T-TF.7 http://illuminations.nctm.org/Lesson.aspx?id=1211 • Human Conics – G-GPE.3 http://illuminations.nctm.org/Lesson.aspx?id=3003 • Foxes and Rabbits: F.TF.5 (2007)	Use the values on a unit circle to generate the graphs of the sine and cosine functions on the coordinate plane. Explore different ways to prove the Law of Sines and Cosines. Derive the Law of Sines from the formula of the area of the non-right triangle. Use properties of difference of two squares to find the modulus. Relate the modulus visually using vectors. Graph complex numbers and identify the magnitude of the complex number, the distance of the complex number from the origin, and the direction of the complex number from the origin. Express complex numbers in polar coordinate form and in rectangular form. Tie measures in special right triangles to values on the unit circle and use those values to generate a relationship between the angles and the corresponding locations on the unit circle. The algebraic proofs for sum and difference formulas for sine and cosine flow nicely once you know the cosine formulas. First use the distance formula and Pythagorean	**Illustrative Mathematics** Properties of Trigonometric Functions: http://www.illustrativemathematics.org/illustrations/1704

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
• http://s3.amazonaws.com/illustrativemathematics/illustration_pdfs/000/000/817/original/illustrative_mathematics_817.pdf?1390750613 **Wolfram Demonstration:** N-CN.4 • http://demonstrations.wolfram.com/ComplexNumber/ **Engage New York** • http://www.engageny.org/sites/default/files/resource/attachments/precalculus-m1-module-overview-and-assessments.pdf : N-CN.4 and N-CN.5 **LAUSD Adopted Textbooks** Precalculus Enhanced with Graphing Utilities, 4th Edition , Sullivan & Sullivan, Pearson/Prentice Hall (2005). Precalculus Graphical, Numerical, Algebraic, 7th edition, Demana, Waits, Foley & Kennedy, Addison Wesley, Pearson Education (2007). Pre-Calculus with Limits: A Graphing Approach, 5th edition, Larson, Hostetler, and Edwards, Houghton/Mifflin, Boston/New York (2008). Precalculus with Trigonometry Concepts and Applications, 2nd edition, Foerster, Key Curriculum	identity to derive the cosine formulas. Then allow students to derive the formulas for sine and tangent. Have students explore the conic sections and describe how to cut a cone to create various conic sections. Import images of circles from fields from Google Earth into a coordinate grid system and find their equations.	

LANGUAGE GOALS

Writing:

1. Students will explain in writing how to prove and apply the Laws of Sines and Cosines using technical vocabulary in complex sentences.
   Example: To derive the Law of Sines from the formula of the area of the non-right triangle, I (draw the altitude) h from the (vertex) A of the triangle from the definition of the (sine function).
2. Students will explain (in writing and orally) the terms and definitions of the trigonometric functions; conic sections; and complex numbers.
   Example: To find the (amplitude) of the function, I can first find the (midline) and then find the distance to the (maximum or minimum) of the graph.
3. Students will compare and contrast in writing the differences between a circle and an ellipse.

LANGUAGE GOALS
Example: I can derive the formula A=1/2 ab sin (C) for the area of a triangle by drawing an (auxiliary line) from a (vertex) that is (perpendicular) to the opposite side. Listening and Speaking: 4. Students will generate class discussions using academic vocabulary related to the rectangular and polar forms of complex numbers. *Example*: Complex number can be expressed in (polar coordinate) form and in (rectangular form) by _____. Reading: 5. Students will read a word problem and identify the language needed to create an algebraic representation in order to solve the problem.
PERFORMANCE TASKS
Pre-Calculus with Limits: A Graphing Approach, 5th edition, Larson, Hostetler, and Edwards, Houghton/Mifflin, Boston/New York, 2008. **F-TF.4**. - Electrical Circuits, #73, page 275; **F-TF.7 –** Photography, page 329 #83; **F-TF.9 –** Standing Waves**,** page 385, #79; Harmonic Motion, page 386, #80, **F-TF.10 –** Railroad Track, page 397, #129; Mach Number, page 398, #128. **G-GPE.3 –** Architecture, page 678, #47- 49; **G-SRT.10 –** Surveying, page 422, #38; Landau Building, page 422, # 45.

DIFFERENTIATION		
FRONT LOADING	ACCELERATION	INTERVENTION
• Involve students to have a discussion that center around extending their knowledge of creating and analyzing systems of linear equations and inequalities. Have them use their prior knowledge of graphing linear equations to approach system of linear and quadratic equations with two variables. • Engage students in an activity that would involve comparing linear equations with quadratics equations, and then quadratics equations and exponential equations. • Have students match linear, quadratic, and exponential functions with their graphs, tables, and equations. • Direct students to connect the idea of functions with trigonometry and see sine,	• Consider using Application Problems found in the textbook for real-world examples that can be solved by writing an equation, and have students explore the graphs of equations using technology application. • Provide examples of real-world problems that can be modeled by writing linear, polynomial, rational, absolute value, exponential, and logarithmic functions. Have students use technology to graph the functions, make tables of values, or find successive approximations resulting from the function. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	• Show students how to create numerical equations and then introduce linear equations in one variable. Students can make comparisons using the numerical and linear equations. • For graphing, have students make a T-chart of the equations, graph them and them analyze, find the intersection of the equations, and then explain what that means. Include a case where they would compare simple linear and quadratics equations, e.g. $y=2x$ and $y=x^2$ • Precalculus intervention should include strategies such as targeted grouping peer and counseling grouping,

DIFFERENTIATION📖		
FRONT LOADING	ACCELERATION	INTERVENTION
cosine, and tangent values as functions of angle values input in radians. • Review the definition of circles as a set of points whose distance from a fixed point is constant. • Review the algebraic method of completing the square. • Illustrate conic sections geometrically as cross sections of a cone. • Have students define conic sections and illustrate it picturically. • If the imaginary unit *i* is misinterpreted as -1 instead of √□1, re-establish a definition of *i*.	• Give students examples of real-world problems that can be solved by writing an equation, and have students explore the graphs of the equations using technology application to determine which parts of the graph are relevant to the problem context. • Have students write a system of two equations in two variables where one equation is quadratic and the other is linear such that the system has no solution. Explain, using graphs, algebra and/or words, why the system has no solution.	• Use informal techniques frequently during regular class time to gauge student understanding. • Use questioning that focuses on student thinking and reasoning to help you monitor your students. • Incorporate writing activities and group work to observe student thinking and identify misconceptions and gaps in understanding. • Have students illustrate concepts using drawings, graphs, and models. • Many students who need intervention struggle to learn concepts because they may not be able to grasp abstract concepts. Whenever possible, vary your instructional techniques to include use of models, manipulatives, and technology.

References:

1. National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards (Mathematics)*. Washington D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers.
2. McCallum, W., Zimba, J., & Daro, P. (2011, December 26 Draft). *Progressions for the Common Core State Standards in Mathematics*. Cathy Kessel (Ed.). Retrieved from http://ime.math.arizona.edu/progressions/#committee.
3. Engage NY. (2012). New York Common Core Mathematics Curriculum. Retrieved from http://www.engageny.org/resource/high-school-algebra-i.
4. Mathematics Assessment Resource Service, University of Nottingham. (2007 - 2012). Mathematics Assessment Project. Retrieved from http://map.mathshell.org/materials/index.php.
5. Smarter Balanced Assessment Consortium. (2012). Smarter Balanced Assessments. Retrieved from http://www.smarterbalanced.org/.
6. Partnership for Assessment of Readiness for College and Career. (2012). PARCC Assessments. Retrieved from http://www.parcconline.org/parcc-assessment.
7. California Department of Education. (2013). Draft Mathematics Framework Chapters. Retrieved from http://www.cde.ca.gov/be/cc/cd/draftmathfwchapters.asp.
8. National Council of Teachers of Mathematics (NCTM) Illuminations. (2013). Retrieved from http://illuminations.nctm.org/Weblinks.aspx.

9. The University of Arizona. (2011-12). Progressions Documents for the Common Core Math Standards. Retrieved from http://ime.math.arizona.edu/progressions.
10. Larson, R.; Hostetler, R.; and Edwards, B. H. (2008). Pre-Calculus with Limits: A Graphing Approach, 5th edition. Boston, New York: Houghton/Mifflin.
11. Sullivan, M. & Sullivan III, M. (2006). *Precalculus Enhanced with Graphing Utilities*, 4th edition. New Jersey: Pearson, Prentice Hall.
12. Demana, F.D. Waits, B.K. Foley, G.D. & Kennedy, D. (2007). *Precalculus Graphical, Numerical, Algebraic*, 7th edition. Addison Wesley, Pearson Education.
13. Foerster, P. A.(2007). *Precalculus with Trigonometry Concepts and Applications*, 2nd edition. Emeryville, CA: Key Curriculum.