Algebra 2 – UNIT 4
Geometry and Trigonometry

Critical Area: Students use algebraic manipulation, including completing the square, as a tool for geometric understanding to determine if the equation represents a circle or a parabola. They graph shapes and relate the graphs to the behavior of the functions with the transformation on the variable (e.g. the graph of y=f (x+2). Students expand on their understanding of the trigonometric functions first developed in Geometry to explore the graphs of trigonometric functions with attention to the connection between the unit circle representation of the trigonometric functions and their properties, use trigonometric functions to model periodic phenomena. Students use Pythagorean identity to find the trig function outputs given the angle and understand that interpretation of sine and cosine yield the Pythagorean Identity.

CLUSTERS	COMMON CORE STATE STANDARDS (*) Indicates a modeling standard linking mathematics to everyday life, work, and decision making.
Translate between the geometric description and the equation for a conic section	**Geometry – Expressing Geometry Properties with Equations** **G-GPE.3.1.** Given a quadratic equation of the form $ax^2 + by^2 + cx + dy + e = 0$, use the method for completing the square to put the equation into standard form; identify whether the graph of the equation is a circle, ellipse, parabola, or hyperbola, and graph the equation. [In Algebra II, this standard addresses circles and parabolas only.] CA
Extend the domain of the trigonometric functions using the unit circle	**Functions – Trigonometric Functions** **F-TF.1.** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **F-TF.2.** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **F-TF.2.1.** Graph all 6 basic trigonometric functions.
Model periodic phenomena with trigonometric functions	**F-TF.5.** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ★
Prove and apply trigonometric identities	**F-TF.8.** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant.
MATHEMATICAL PRACTICES	
1. **Make sense of problems and persevere in solving them.** 2. **Reason abstractly and quantitatively.** 3. **Construct viable arguments and critique the reasoning of others.**	Emphasize all the mathematical practice standards as you address the standards in this unit. F-TF.5 would provide the opportunity to link mathematics to everyday life, work, and decision making.

4. Model with mathematics. **5. Use appropriate tools strategically.** **6. Attend to precision.** **7. Look for and make use of structure.** 8. **Look for and express regularity in repeated reasoning.**	

LEARNING PROGRESSIONS

High School Progression on Functions
http://commoncoretools.me/wp-content/uploads/2013/07/ccss_progression_functions_2013_07_02.pdf

★Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.
(+) Indicates additional mathematics to prepare students for advanced courses.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• A circle is a set of points that can be defined by an equation. This measurement is expressed in radians rather than degrees. • Students extend the domain of trigonometric functions using the unit circle. • Students establish a way to measure angles with respect to arc length. • The trigonometric functions are extended to all real numbers to describe rotations around the unit circle. • Our world is periodic. The amount of sunlight a city receives on a given day, high and low tides are all real life instances where sinusoids explain and model real life phenomena. • Prove the Pythagorean identity $\sin^2(\Theta) + \cos^2(\Theta) = 1$ and use it to find $\sin(\Theta)$, $\cos(\Theta)$, or $\tan(\Theta)$ given $\sin(\Theta)$, $\cos(\Theta)$, or $\tan(\Theta)$ and the quadrant of the angle. • The Theorem of Pythagoras comes up in many places, including trigonometry, through the use of standard graphing form of a circle. It is used to prove many trigonometric identities.	• How do you write the equation of a circle? • What is the angle of rotation, and how is it measured? • How can you explain the unit circle? • Why do we need radian measure? • Why are radians said to be unitless measures of angles? • How can sine, cosine, and tangent functions be defined using the unit circle? • What are periodic functions? • Why is modeling them so important? • Why is the Theorem of Pythagoras so essential?	amplitude angle of rotation completing the square cosecant function cosine function cotangent function coterminal dilation domain initial side intercepted arc midline periodic function phase shift quadrants radian measure range reference angles secant function sine function sinusoid special right triangles standard form of a circle standard position tangent function

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
		terminal side translation unit circle

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
LAUSD Adopted Textbooks and Programs • Big Ideas Learning - Houghton Mifflin Harcourt, 2015: Big Ideas Algebra 2 • College Preparatory Mathematics, 2013: Core Connections, Algebra 2 • The College Board, 2014:Springboard Algebra 2 **Illustrative Mathematics Resources**: • Explaining the equation for a circle:G-GPE.3 http://www.illustrativemathematics.org/illustrations/1425 • Foxes and Rabbits 3: F-TF.5 http://www.illustrativemathematics.org/illustrations/817 • Trig Functions and the Unit Circle : F-TF.2 https://www.illustrativemathematics.org/illustrations/1820 **NCTM Illuminations** • Graphs from the Unit Circle: F-TF.1, 2 http://illuminations.nctm.org/LessonDetail.aspx?id=L785	Show students how to find sine, cosine, and tangent values by constructing right triangles on a Cartesian plan, measuring the lengths of the sides, and computing the ratios. Using graphing calculators or computer software, as well as graphing simple examples by hand, have students graph a variety of trigonometric functions in which the amplitude, frequency, and/or midline is changed. Students should be able to generalize about parameter changes, such as what happens to the graph of $y = \sin(x)$ when the equation is changed to $y = 2\sin(x) + 5$. Use graph paper and paper plate to model trigonometric functions Use teacher-guided comparison conversations to ensure that students are able to make connections Mathematics Journal Sample Prompts: • What patterns did you find in ….? • How do you …? • Review what you did today and explain how it is similar to something you already knew? • Is there a shortcut for finding …? What is it? How does it work? Why does it work? Emphasize multiple representations in teaching new and review older vocabulary: • Words • Algebraically • Tables of Data	**Formative Assessment** **LAUSD Assessments** The district will be using the SMARTER Balanced Interim Assessments. Teachers would use the Interim Assessment Blocks (IAB) to monitor the progress of students. Each IAB can be given twice to show growth over time. **State Assessments** California will be administering the SMARTER Balance Assessment as the end of course for grades 3-8 and 11. There is no assessment for Algebra 1. The 11th grade assessment will include ítems from Algebra 1, Geometry, and Algebra 2 standards. For examples, visit the SMARTER Balance Assessment at: http://www.smarterbalanced.org/

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
	• Graphically • Symbolically Use a compass and straightedge to explore a unit circle with a fixed radius of 1. Help students to recognize that the circumference of the circle is 2π, which represents the number of radians for one complete revolution around the circle. Students can determine that, for example, $\pi/4$ radians would represent a revolution of 1/8 of the circle or 45°. Students can examine how a counterclockwise rotation determines a coordinate of a particular point in the unit circle from which sine, cosine, and tangent can be determined. Have students explore real-world examples of periodic functions; such as: average high (or low) temperatures throughout the year, the height of ocean tides as they advance and recede, and the fractional part of the moon that one can see on each day of the month. Graphing some real-world examples can allow students to express the amplitude, frequency, and midline of each.	

LANGUAGE GOALS for low achieving, high achieving, students with disabilities and English Language Learners
• Students will explain orally and in writing the attributes specific of a given quadratic equation of the form $ax^2 + by^2 + cx + dy + e = 0$, and identify whether the graph is [of] a circle or an a parabola. *Example: "*$3x^2 + x + y + 5 = 0$ is a parabola because the coefficient in front of y^2 is zero. • Students will listen for and point out similarities in their classmates' ideas using the words *similar, identical,* and *alike*. *Example:* _____'s ideas were similar/identical/alike to _____'s idea. • Students will explain the differences and similarities between trigonometric functions, using the following specific set of words: *sine, cosine, tangent, period,* and *amplitude.* • *Example*: " y = sin x and y = cos x graphs both have period of 360^o (2π) and an amplitude of 1"
PERFORMANCE TASKS
Illustrative Mathematics • As the Wheel Turns: F-TF.5, F-IF http://www.illustrativemathematics.org/illustrations/595

- Foxes and Rabbits 2: F-TF.5 http://www.illustrativemathematics.org/illustrations/816

Mathematics Assessment Project (MARS Tasks)

- The Ferris Wheel:F-TF.5 http://map.mathshell.org/materials/download.php?fileid=1252

NCTM Illuminations Lessons

- The Unit Circle: F-TF.1, 2 http://illuminations.nctm.org/LessonDetail.aspx?id=L785
- Hanging Chains: G-GPE.3.1 http://illuminations.nctm.org/LessonDetail.aspx?id=L628
- Rolling into Radians: http://illuminations.nctm.org/LessonDetail.aspx?id=L844
- Seeing Music: F-TF.5 http://illuminations.nctm.org/LessonDetail.aspx?id=L686
- Graphing Trigonometric Functions: F-TF.2.1 and F-TF.5 http://illuminations.nctm.org/ActivityDetail.aspx?ID=174

DIFFERENTIATION

FRONT LOADING	ACCELERATION	INTERVENTION
In the extension of the trigonometric functions to the unit circle, proficient students must use repeated reasoning (MP.8). Students will model real world situations with trigonometric functions (MP.4). Use of trigonometric vocabulary, such as (amplitude, frequency, period, midline, degree, and radian) aid in communicating precisely (MP.6). **Pre-Teach Vocabulary** • State the definitions, and have students repeat the definitions. • Provide students with correct and incorrect usage of the word • Equation of Circles 1: http://map.mathshell.org/materials/download.php?fileid=1202	Students can investigate daylight hours model and many other trigonometric modeling situations such as simple predator-prey models, sound waves, and noise cancellation models. Engage students to use trigonometric functions to model periodic phenomena. Connected to standard F-BF.3 (families of functions), they begin to understand the relationship between the parameters appearing in the general cosine function $f(x)=A\cdot\cos(Bx-C)+D$ (and sine function) and the graph and behavior of the function (e.g., amplitude, frequency, line of symmetry).	Reteach the trigonometry ratio and remind students how to use (SOHCAHTOAH) to remember the trigonometric ratios. Teach students how to graph all 6 basic trigonometric functions, namely: sine, cosine, tangent, cotangent, secant, and cosecant. They can use any graphing utility such as graphing calculator, apps, and graphing software to graph the families of functions. Have students analyze and explain the meaning of amplitude, frequency, period, and midline based on their graphs.

References:

1. National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards (Mathematics)*. Washington D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers.
2. McCallum, W., Zimba, J., Daro, P. (2011, December 26 Draft). *Progressions for the Common Core State Standards in Mathematics*. Cathy Kessel (Ed.). Retrieved from http://ime.math.arizona.edu/progressions/#committee.
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4. Mathematics Assessment Resource Service, University of Nottingham. (2007 - 2012). Mathematics Assessment Project. Retrieved from http://map.mathshell.org/materials/index.php.
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7. California Department of Education. (2013). Draft Mathematics Framework Chapters. Retrieved from http://www.cde.ca.gov/be/cc/cd/draftmathfwchapters.asp.
8. National Council of Teachers of Mathematics (NCTM) Illuminations. (2013). Retrieved from http://illuminations.nctm.org/Weblinks.aspx.
9. The University of Arizona. (2011-12). Progressions Documents for the Common Core Math Standards. Retrieved from http://ime.math.arizona.edu/progressions.