Algebra 2 – UNIT 3
FUNCTIONS

Critical Area: Instructional time should focus on relating arithmetic of rational expressions to arithmetic of rational numbers. Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations. Students will expand understandings of functions and graphing to include trigonometric functions. Building on their previous work with functions and on their work with trigonometric ratios and circles in the Geometry course, students now use the coordinate plane to extend trigonometry to model periodic phenomena. Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function.

CLUSTERS	COMMON CORE STATE STANDARDS
(m) Interpreting functions that arise in applications in terms of the context **(m) Analyze Functions Using Different Representations**	**Functions – Interpreting Functions** **F-IF.4**. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ★ **F-IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. ★ **F-IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★ **F-IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★ b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. ★ c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. ★ e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. ★ **F-IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **F-IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
(m) Build a function that models a relationship between two quantities **(s) Build new functions from existing functions**	**Functions – Building Functions** **F-BF.1** Write a function that describes a relationship between two quantities.★ b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*★

	F-BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* **F-BF.4** Find inverse functions. a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) = 2x^3$ or $f(x) = (x + 1)/(x - 1)$ for $x \neq 1$*
(s) Construct and compare linear, quadratic, and exponential models and solve problems	**Functions – Linear, Quadratic, and Exponential Models** **F-LE.4** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. ★[Logarithms as solutions for exponentials.] **4.1 Prove simple laws of logarithms. CA ★** **4.2 Use the definition of logarithms to translate between logarithms in any base. CA ★**
(s) Perform arithmetic operations with complex numbers **(s) Use complex numbers in polynomial identities and equations.** [Polynomials with real coefficients.]	**Number and Quantity – Complex Number System** **N-CN.1**.Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. **N-CN.2**. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. **N-CN.7** Solve quadratic equations with real coefficients that have complex solutions. **N-CN.8** (+) Extend polynomial identities to the complex numbers. *For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.* **N-CN.9** (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
MATHEMATICAL PRACTICES	LEARNING PROGRESSIONS
1. **Make sense of problems and persevere in solving them.** 2. **Reason abstractly and quantitatively.** 3. Construct viable arguments and critique the reasoning of others. 4. **Model with mathematics.** 5. **Use appropriate tools strategically.** 6. **Attend to precision.** 7. **Look for and make use of structure.** 8. Look for and express regularity in repeated reasoning.	Emphasize Mathematics Practices 1, 2, 4, 5, and 7 in this unit.
LEARNING PROGRESSIONS	
High School Progression on Functions http://commoncoretools.me/wp-content/uploads/2013/07/ccss_progression_functions_2013_07_02.pdf	

(m) Major Clusters – area of intensive focus where students need fluent understanding and application of the core concepts.
(S) Supporting/Additional Clusters – designed to support and strengthen areas of major emphasis/expose students to other subjects.
★Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.
(+) Indicates additional mathematics to prepare students for advanced courses.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• Functions need to be understood and interpreted in terms of their context. • A function can be represented in different ways; these different representations help with analysis of the function. • A function can be used to model the relationship between two quantities. • New functions from existing functions can be understood and built. • Comparing linear, quadratic and exponential models to solve problems is understood and constructed. • Real and complex numbers are important in solving and understanding polynomial equations.	1. What is a function and how does it model a relationship between two quantities? 2. How would you write a function that describes a relationship between two quantities? 3. What are the differences and similarities between real and complex solutions of polynomial equations? Explain graphically or algebraically. 4. How do you differentiate between an exponential and a logarithmic function? 5. How and when do we use laws of logarithms?	• absolute value function • complex numbers • complex roots • end behavior • function • Exponential • interpret • inverse function • Laws of Logarithms • logarithmic • periodicity • piecewise function • relation • relative Maximums • relative Minimum • step function • symmetries • transformations • trigonometric

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
LAUSD Adopted Textbooks and Programs • Big Ideas Learning - Houghton Mifflin Harcourt, 2015: Big Ideas Algebra 2 • College Preparatory Mathematics, 2013: Core Connections, Algebra 2 • The College Board, 2014:Springboard Algebra 2 **Illustrative Mathematics** Bacteria Populations: F-LE.4 http://www.illustrativemathematics.org/illustrations/370	Students can make use of graphing software to investigate the effects of replacing a function (x) by $(x)+k$, $kf(x)$, $f(kx)$, and $f(x+k)$ for different types of functions (MP.5). Tables and graphs should be used to support student understanding of F-BF-4a. This standard dovetails with standard F-LE-4 and should be taught in progression with it. Students understand logarithms as functions that *undo* their corresponding exponential functions;	**Formatiive Assessment** PARCC - http://www.parcconline.org/samples/mathematics/high-school-mathematics

Running Time: F-IF.7c http://www.illustrativemathematics.org/illustrations/1539 Graphs of Power Functions: F-IF.7c http://www.illustrativemathematics.org/illustrations/627 Exponentials and Logarithms I: F-BF.4 http://www.illustrativemathematics.org/illustrations/600 Exponentials and Logarithms II: F-BF.5, F-LE.4 http://www.illustrativemathematics.org/illustrations/615 Complex number patterns: N-CN.1 http://www.illustrativemathematics.org/illustrations/722 Powers of a complex number: N-CN.2 http://www.illustrativemathematics.org/illustrations/1689 Completing the square: N-CN.7; A-REI.4 http://www.illustrativemathematics.org/illustrations/1690 **Inside Mathematics** Measuring Mammals- F-BF.4 http://www.insidemathematics.org/problems-of-the-month/pom-measuringmammals.pdf	opportunities for instruction should emphasize this relationship.	**LAUSD Assessments** The district will be using the SMARTER Balanced Interim Assessments. Teachers would use the Interim Assessment Blocks (IAB) to monitor the progress of students. Each IAB can be given twice to show growth over time. **State Assessments** California will be administering the SMARTER Balance Assessment as the end of course for grades 3-8 and 11. There is no assessment for Algebra 1. The 11th grade assessment will include ítems from Algebra 1, Geometry, and Algebra 2 standards. For examples, visit the SMARTER Balance Assessment at: http://www.smarterbalanced.org/

LANGUAGE GOALS for low achieving, high achieving, students with disabilities and English Language Learners

- Students will describe orally and in writing the process of graphing functions using the terms: intercepts, end behavior, and maximum/minimum.
- Students will describe orally and in writing transformations in terms of the parent function.
- Students will compare and contrast functions and their inverses in oral and written form.
- Students will describe orally and in writing the steps required to express exponential equations in logarithmic form and solve them.
- Student will describe orally and writing how they apply polynomial operations to complex numbers; *for example,* $x^2 + 4 = (x + 2i)(x - 2i)$.

PERFORMANCE TASKS

Mathematics Assessment Project Formative Assessments/Tasks

Patchwork – F-BF.1 http://map.mathshell.org/materials/download.php?fileid=754

Sidewalk Patterns – F-BF.1 http://map.mathshell.org/materials/download.php?fileid=760

Printing Tickets – F-IF.4 http://map.mathshell.org/materials/download.php?fileid=772

Illustrative Mathematics

Identifying graph of functions – F.IF.7c

http://www.illustrativemathematics.org/illustrations/803

Inside Mathematics

Digging Dinosaurs- F-IF.8, F-LE.1 http://www.insidemathematics.org/problems-of-the-month/pom-diggingdinosaurs.pdf

DIFFERENTIATION📖		
UDL/FRONT LOADING	ACCELERATION	INTERVENTION
Have students discuss the relationship between a function and definition of its inverse. Engage students in an activity to graph square root, Cube root, and piecewise-defined functions, including step functions and absolute value functions. Have students practice how to graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.	Activate a discussion around explicit and implicit function. Students could use a T-chart to describe the differences between explicit and implicit function. Have students show symbolically and graphically how to transform a function to its inverse. Show that when f(x)=2x+3, then x=2f(x)+3. Solving for f(x), the resulting inverse will be f(x)=(x-3)/2 Example: y=2x+3, then x=2y+3 solve for y gives the inverse of (x) as y=(x-3)/2. Graphically you can show the inverse of a function. Make a table of the function f(x) and its inverse. Graph x and f(x) values from the table and then develop another table by switching the numbers.	Student use apps, software, or graphing calculator to practice graphing square root, Cube root, and piecewise-defined functions, including step functions and absolute value functions. They describe the behavior of each of the above functions and write about them.

References:

1. National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards (Mathematics)*. Washington D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers.
2. McCallum, W., Zimba, J., Daro, P. (2011, December 26 Draft). *Progressions for the Common Core State Standards in Mathematics*. Cathy Kessel (Ed.). Retrieved from http://ime.math.arizona.edu/progressions/#committee.
3. Engage NY. (2012). New York Common Core Mathematics Curriculum. Retrieved from http://www.engageny.org/sites/default/files/resource/attachments/algebra-ii-m1-module-overview-and-assessments.pdf.
4. Mathematics Assessment Resource Service, University of Nottingham. (2007 - 2012). Mathematics Assessment Project. Retrieved from http://map.mathshell.org/materials/index.php.
5. Smarter Balanced Assessment Consortium. (2012). Smarter Balanced Assessments. Retrieved from http://www.smarterbalanced.org/.
6. Partnership for Assessment of Readiness for College and Career. (2012). PARCC Assessments. Retrieved from http://www.parcconline.org/parcc-assessment.
7. California Department of Education. (2013). Draft Mathematics Framework Chapters. Retrieved from http://www.cde.ca.gov/be/cc/cd/draftmathfwchapters.asp.
8. National Council of Teachers of Mathematics (NCTM) Illuminations. (2013). Retrieved from http://illuminations.nctm.org/Weblinks.aspx.
9. The University of Arizona. (2011-12). Progressions Documents for the Common Core Math Standards. Retrieved from http://ime.math.arizona.edu/progressions.