

Universal Access Chapter

of the

Mathematics Framework

*for California Public Schools:
Kindergarten Through Grade Twelve*

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Table UA-1. Mathematical Language That May Cause Difficulties for English Learners

Words whose meanings are found only in mathematics (used only in academic English)	Hypotenuse, parallelogram, coefficient, quadratic, circumference, polygon, polynomial
Symbolic language (used almost universally)	$+$, $-$, \times , \div , π , $\frac{1}{2}$
Words with multiple meanings in everyday English	<p>The floor is <u>even</u>.</p> <p>The picture is <u>even</u> with the window.</p> <p>Breathing develops an <u>even</u> rhythm during sleep.</p> <p>The dog has an <u>even</u> temperament.</p> <p>I looked sick and felt <u>even</u> worse.</p> <p><u>Even</u> a three-year-old child knows the answer.</p>
Words with multiple meanings in academic English	<p>Number: <u>Even</u> numbers (e.g., 2, 4, 6, and so on)</p> <p>Number: <u>Even</u> amounts (e.g., even amounts of sugar and flour)</p> <p>Measurement: An <u>even</u> pound (i.e., an exact amount)</p> <p>Function: An <u>even</u> function (e.g., $f(x) = f(-x)$ or cosine function)</p>
Phonologically similar words.	<p><i>tens</i> versus <i>tenths</i></p> <p><i>sixty</i> versus <i>sixteen</i></p> <p><i>sum</i> versus <i>some</i></p> <p><i>whole</i> versus <i>hole</i></p> <p><i>off</i> versus <i>of</i></p> <p>How many halves do you have?</p> <p><i>then</i> versus <i>than</i></p>

Adapted from Asturias 2010.

Helping all students meet mathematical language demands requires careful planning; attention to the language demands of each lesson, unit, and module; and ongoing monitoring of students' understanding and their ability to communicate what they know and can do. As students explore mathematical concepts, engage in discussions about mathematics topics, explain their reasoning, and justify their procedures and conclusions, the mathematics classroom will be vibrant with conversation.

disabilities. Accommodations change how a student learns material, and modifications change what a student is taught or expected to learn.

- *Assistive technology accommodations.* Assistive technology provides access to the course curriculum. Students can receive assistance from a computer that scans and reads text or digital content to incorporate images, sound, video clips, and additional information. Students with visual impairments can gain access to instructional materials through digital large print with a contrasting background, the ability to change the font as it appears on the screen, or text-to-speech devices. Software that converts text to braille characters, using a refreshable display, provides students with access to printed information. Students can use mobile devices to create or record notes so that they can later print out assignments or use the notes to study for a test. A student with motor difficulties might use an enlarged or simplified computer keyboard, a talking computer with a joystick, or other modified input device such as a switch, headgear, or eye selection devices. The American Speech–Language–Hearing Association (ASHA) presents information on augmentative and alternative communication systems or applications⁴ that help students with severe speech or language disabilities express thoughts, needs, or ideas. These and other types of assistance can provide access, but they do not change content and are therefore considered accommodations.
- *Assistive technology modifications.* Assistive technology provides additional help to students who otherwise would not be able learn a concept or show what they have learned. Examples of modifications provided by assistive technology include the use of speech-to-text devices, calculators, or other devices that provide information not otherwise available to students. Of course, there are many other types of modifications that do not involve the use of assistive technology.

Although assistive technology helps to level the playing field for students with special needs, many types of assistive technology (both software and hardware) are beneficial for all students. The flexibility of assistive technology allows a teacher to use tools and materials that support a student’s individual strengths and also address his or her disability in the least restrictive environment.

The CDE provides information that clarifies basic requirements for consideration and provision of assistive technology and services to individuals with disabilities. Information is also available for local educational agencies, particularly members of IEP teams, to effectively address these requirements. For other examples of assistive technology, please visit the CDE Assistive Technology Checklist Web page at <http://www.cde.ca.gov/sp/se/sr/atexmpl.asp> (CDE 2015a).

Planning Instruction for California’s English Learners

Students in California demonstrate a wide variety of skills, abilities, and interests as well as different levels of proficiency in English and other languages. California’s students come from diverse cultural, linguistic, ethnic, and religious backgrounds, have different experiences, and live with different familial and socioeconomic circumstances. The greater the variation of the student population, the richer the learning experiences for all, and the more assets upon which teachers may draw. At the same time,

4. For more information about augmentative and alternative communication, visit <http://www.asha.org/public/speech/disorders/AAC/> (ASHA 2015).

the teacher’s role in providing high-quality curriculum and instruction that is sensitive to the needs of individuals becomes more complex. In diverse settings, the notion of *shared responsibility* is particularly crucial. Teachers need the support of one another, administrators, specialists, and the community in order to best serve all students.

Approximately 25 percent of California’s public school students are learning English as an additional language. These students come to California schools from all over the world, but the majority were born in California. Schools and districts are responsible for ensuring that all English learners have full access to an intellectually rich and comprehensive curriculum, via appropriately designed instruction, and that they make steady—and even accelerated—progress in their English language development.

English learners come to school with a range of cultural and linguistic backgrounds; experiences with formal schooling; proficiency with mathematics, their native language, and English; migrant and socioeconomic statuses; and interactions in the home, school, and community. All of these factors inform how educators support English learners to achieve school success through the implementation of the CA ELD standards in tandem with the CA CCSSM. Educators should not confuse students’ language ability with their mathematical understanding.

Ethnically and racially diverse students make up approximately 74 percent of California’s student population, making it the most diverse student population in the nation. In 2012–13, more than 1.3 million students—or roughly 25 percent of the California public school population—were identified as English learners. Of those English learners, 84.6 percent identified Spanish as their home language. The next largest group of English learners, 2.3 percent, identified Vietnamese as their home language (CDE 2013c). Given the large number of English learners in California’s schools, it is essential to provide these students with effective mathematics instruction.

English learners face a significant challenge in learning subject-area content while simultaneously developing proficiency in English. Planning mathematical instruction for English learners is most effective when the instruction takes into consideration the students’ mathematics skills and understandings as well as their assessed levels of proficiency in English and their primary language. Because of variations in academic background and age, some students may advance more quickly in mathematics or English language development than other students who require more support to make academic progress. Many districts use assessment tools such as the statewide assessment,⁵ which measures the progress of English learners in acquiring the skills of listening, speaking, reading, and writing in English. The statewide assessment is designed to identify a student’s proficiency level in English and to monitor the student’s progress in English language development. Other tools for measuring progress in English language development are academic progress, teacher and parent evaluation, and tests of basic skills (such as district benchmarks).

The role of English language proficiency must be a consideration for English learners who experience difficulties in learning mathematics. Even students who have good conversational English skills may lack the academic language necessary to fully access mathematics curriculum (Francis et al. 2006a).

5. This statewide assessment was formerly known as the California English Language Development Test (CELDT) and will be replaced by the English Language Proficiency Assessments for California (ELPAC) in 2016.

Academic language, as described by Saunders and Goldenberg, “entails all aspects of language from grammatical elements to vocabulary and discourse structures and conventions” (Saunders and Goldenberg 2010, 106).

Moschkovich (2012b) cautions that communicating in mathematics is more than a matter of learning vocabulary; students must also be able to participate in discussions about mathematical ideas, make generalizations, and support their claims. She states, “While vocabulary is necessary, it is not sufficient. Learning to communicate mathematically is not merely or primarily a matter of learning vocabulary” (Moschkovich 2012b, 18). Providing instruction that focuses on teaching for understanding, helping students use multiple representations to comprehend mathematical concepts and explain their reasoning, and supporting students’ communication about mathematics is challenging (Moschkovich 2012a, 1). Moschkovich’s recommendations for connecting mathematical content to language are provided in table UA-3.

“[E]very teacher must incorporate into his or her curriculum instructional support for oral and written language as it relates to the mathematics standards and content. It is not possible to separate the content of mathematics from the language in which it is discussed and taught.”

—Francis et al. 2006a, 38

Table UA-3. Recommendations for Connecting Mathematical Content to Language

1. Focus on students’ mathematical reasoning, not accuracy in using language.
2. Shift to a focus on mathematical discourse practices; move away from simplified views of language.
3. Recognize and support students to engage with the complexity of language in math classrooms.
4. Treat everyday language and experiences as resources, not as obstacles.
5. Uncover the mathematics in what students say and do.

Source: Moschkovich 2012a, 5–8.

Teachers can take the following steps to support English learners in the acquisition of mathematical skills and knowledge as well as academic language:

- Explicitly teach academic vocabulary for mathematics, and structure activities in which students regularly employ key mathematical terms. Be aware of words that have multiple meanings (such as *root*, *plane*, *table*, and so forth).
- Provide communication guides, sometimes called *sentence frames*, as a temporary scaffold to help students express themselves not just in complete sentences but articulately within the MP standards.
- Use graphic organizers and visuals to help students understand mathematical processes and vocabulary.

For English learners who are of elementary-school age, progress in mathematics may be supported through intentional lesson planning for content, mathematical practice, and language objectives. Language objectives “articulate for learners the academic language functions and skills that they need to master to fully participate in the lesson and meet the grade-level content standards” (Echevarria, Vogt, and Short 2008). In mathematics, students’ use of the MP standards requires students to translate between various representations of mathematics and to develop a command of receptive (listening, reading) and productive (speaking, writing) language. Language is crucial for schema-building; learners construct new understandings and knowledge through language, whether unpacking new learning for themselves or justifying their reasoning to a peer.

The following are examples of possible language objectives for a student in grade two:

- Read word problems fluently.
- Explain in writing the strategies used to solve addition and subtraction problems within 100.
- Describe orally the relationship between addition and subtraction.

Francis et al. (2006a) examined research on instruction and intervention in mathematics for English learners. The consensus among the researchers was that a lack of development of academic language is a primary cause of English learners’ academic difficulties and that more attention needs to be paid to the development of academic language. Like Moschkovich, Francis et al. (2006a) make clear that understanding and using academic language involve many skills beyond merely learning new vocabulary words; these skills include using increasingly complex words, comprehending and using sentence structures and syntax, understanding the organization of text, and producing writing appropriate to the content and to the students’ grade level.

One approach to improve students’ academic language is to “amplify, rather than simplify” new vocabulary and mathematical terms (Wilson 2010). When new or challenging language is continually simplified for English learners, they cannot gain the academic language necessary to learn mathematics. New vocabulary, complex text, and the meanings of mathematical symbols need to be taught in context with appropriate scaffolding or amplified. Amplification helps increase students’ vocabulary and makes mathematics more accessible to students with limited vocabulary. In the progression of rational-number learning throughout the grades, particularly relevant to upper elementary and middle school, students encounter increasingly complex uses of mathematical language (words, symbols) that may contradict student sense-making and associations of a term or phrase from earlier grades. For example, *half* is interpreted as either a call to divide a certain quantity by two, or to double that quantity, depending upon the context:

Half of 6 is _____?

Six (6) divided by one-half is _____?

The standards distinguish between *number* and *quantity*, where quantity is a numerical value of a specific unit of measure. By middle school, students are expected to articulate that a “unit rate for Sandy’s bike ride is one-half mile per hour,” based upon reading the slope of a distance-versus-time line graph

of a bike ride traveled at this constant rate. Here, “one-half” represents the distance traveled for each hour, rather than the equivalent ratio of one mile traveled for every two hours. The same symbols that students encountered in early elementary grade levels to represent parts of a whole—for example, partitioning in grade two, formalized unit fractions in grade three—are now attached to new language and concepts in upper elementary grade levels and middle school.

Mathematical Discourse

According to the New Zealand Council for Educational Research (2014), “Mathematical classroom discourse is about whole-class discussions in which students talk about mathematics in such a way that they reveal their understanding of concepts. Students also learn to engage in mathematical reasoning and debate.” Teachers ask “strategic questions that elicit from students both how a problem was solved and why a particular method was chosen” (New Zealand Council for Educational Research 2014). Students learn to critique ideas (their own and those of other students), and they look for efficient mathematical solutions.

Researchers caution that focusing on academic language alone may promote teaching vocabulary without a context or lead to the misconception that students are lacking because of their inability to use academic language (Edelsky 2006; MacSwan and Rolstad 2003). It is essential for instruction to include teaching vocabulary in context so that the mathematical meaning can be emphasized. Classroom discourse is one instructional strategy that promotes the use of academic and mathematical language within a meaningful context. *Mathematics discourse* is defined as communication that centers on making meaning of mathematical concepts; it is more than just knowing vocabulary. It involves negotiating meanings by listening and responding, describing understanding, making conjectures, presenting solutions, challenging the thinking of others, and connecting mathematical notations and representations (Celedón-Pattichis and Ramirez 2012, 20).

Lesson plans that include objectives for language, mathematical content standards, and mathematical practice standards need to identify where these three objectives intersect and what specific scaffolds are necessary for English learners’ mathematical discourse. As one example, a high school teacher of long-term English learners has planned a lesson that requires students to identify whether four points on a coordinate graph belong to a quadratic or an exponential function. Classroom routines for partner and group work have been established, and students know what “good listening” and “good speaking” look like and sound like. However, the teacher has also created bookmarks for students to use, with sentence starters and sentence frames to share their conjectures and rationales and to question the thinking of other students. The teacher is employing an instructional strategy called “Think-Write-Pair-Share” with scaffolds in the form of sentence frames. After a specified time for individual thinking and writing, students share their initial reasoning with a partner. A whole-class discussion ensues, with the teacher intentionally re-voicing student language and asking students to use their own words to share what they heard another student say. While the teacher informally assesses how students employ academic language in their oral statements, she also presses for “another way to say” or represent that thinking to amplify academic language.

Long-Term English Learners

The lack of English language proficiency and understanding of the language of mathematics is of particular concern for long-term English learners—students in grades six through twelve who have been enrolled in American schools for more than six years and have remained at the same English language proficiency level for two or more consecutive years, as determined by the state’s annual English language development test. To address the instructional needs of long-term English learners, focused instruction such as instructed English language development (ELD) may be the most effective (Dutro and Kinsella 2010). Instructed ELD, as described by Dutro and Kinsella, focuses attention on language learning. Language skills are taught in a prescribed scope and sequence, ELD is explicitly taught, and there are many opportunities for student practice. Lessons, units, and modules are designed to build fluency and aim to help students achieve full English proficiency.

In addition to systematic ELD instruction, Dutro and Moran (2003) offer two recommendations for developing students’ language in the content areas: front-loading and using “teachable moments.”

Front-loading of ELD describes a focus on language preceding a content lesson. The linguistic demands of a content task are analyzed and taught in an up-front investment of time to render the content understandable to the student. This front-loading refers not only to the vocabulary, but also to the forms or structures of language needed to discuss the content. The content instruction, like the action of a piston, switches back and forth from focus on language, to focus on content, and back to language. (Dutro and Moran 2003, 4)

The following example of Dutro and Moran’s “piston” instructional strategy informally assesses and advances students’ mathematical English language development.

List-Group-Label Activity

Purpose: Formative assessment of students’ acquisition of academic language, as well as their ability to distinguish form and function of mathematical terms and symbols. For example, the term *polygon* reminds students of types of polygons (triangles, rectangles, rhombuses) or reminds students of components or attributes of polygons (angles, sides, parallel, perpendicular) or non-examples (circles).

Process: At the conclusion of instruction, the teacher posts a mathematical category or term that students encountered in the unit and asks students to generate as many mathematical words or symbols related to the posted term as they can.

Working with a partner or group, students compile lists of related words and agree how to best sort their lists into subgroups.

For each subgroup of terms or symbols, students must come to agreement on an appropriate label for the subgroup list and be prepared to justify their “List-Group-Label” to another student group.

Teachers also take advantage of teachable moments to expand and deepen language skills. Teachers must utilize opportunities “as they present themselves, to use precise language [MP.6] to fill a specific, unanticipated need for a word or a way to express a thought or idea. Fully utilizing the teachable

moment means providing the next language skill needed to carry out a task or respond to a stimulus” (Dutro and Moran 2003, 4).

M. J. Schleppegrell (2007) agrees that the language of mathematical reasoning differs from informal ordinary language. Traditionally, teachers have identified mathematics vocabulary as a challenge but are not aware of the grammatical patterning embedded in mathematical language that generates difficulties. Schleppegrell identifies these linguistic structures as “patterns of language that draw on grammatical constructions that create dense clauses linked with each other in conventionalized ways” (Schleppegrell 2007, 146) yet differ from ordinary use of language. Examples include the use of long, dense noun phrases such as *the volume of a rectangular prism with sides 8, 10, and 12 cm*; classifying adjectives that precede the noun (e.g., *prime number, right triangle*); and qualifiers that come after the noun (e.g., *a number that can be divided by 1 and itself*). Other challenging grammatical structures that may pose difficulty include signal words such as *if, when, therefore, given, and assume*, which are used differently in mathematics than in everyday language (Schleppegrell 2007, 143–146). Schleppegrell asserts that educators need to expand their knowledge of mathematical language to recognize when and how to include grammatical structures that enable students to participate in mathematical discourse.

Other work on mathematics discourse, such as from Suzanne Irujo (cited in Anstrom et al. 2010), provides concrete classroom applications for vocabulary instruction at the elementary and secondary levels. Irujo explains and suggests three steps for teaching mathematical and academic vocabulary (Anstrom et al. 2010, 23):

- The first suggested step is for educators to analytically read texts, tests, and materials to identify potential difficulties, focusing on challenging language.
- The second step follows Dutro and Moran’s findings on pre-teaching with experiential activities in mathematics; only the necessary vocabulary and key concepts are taught to introduce the central ideas.
- The third and final step is integration of the learning process. New vocabulary is pointed out as it is encountered in context, its use is modeled frequently by the teacher, and the modeling cycle is repeated, followed by guided practice, small-group practice, and independent practice. Irujo also recommends teaching complex language forms (e.g., prefixes and suffixes) through mini-lessons.

Despite the importance of academic language for success in mathematics, “in mathematics classrooms and curricula the language demands are likely to go unnoticed and unattended to” (Francis et al. 2006a, 37). Both oral and written language need to be integrated into mathematics instruction. All students, not just English learners, must be provided many opportunities to engage in mathematics discourse—to talk about mathematics and explain their reasoning. The language demands of mathematics instruction must be noted and attended to. Mathematics instruction that includes reading, writing, and speaking enhances students’ learning. As lessons, units, and modules are planned, both language objectives and content objectives should be identified. By focusing on and modifying instruction to address English learners’ academic language development, teachers support their students’ mathematics learning.

The CA ELD standards are an important tool for designing instruction to support students’ reading, writing, speaking, and listening in mathematics. The CA ELD standards help guide curriculum, instruction,

and assessment for English learners who are developing the English language skills necessary to engage successfully with mathematics. California's English learners (ELs) are enrolled in a variety of school and instructional settings that influence the application of the CA ELD standards. The CA ELD standards are designed to be used by all teachers of academic content and of English language development in all settings, albeit in ways that are appropriate to each setting and to identified student needs. Additionally, the CA ELD standards are designed and intended to be used *in tandem with* the CA CCSSM to support ELs in mainstream academic content classrooms.

Neither the CA CCSSM nor the CA ELD standards should be treated as checklists. Instead, the CA ELD standards should be utilized as a tool to equip ELs to better understand mathematics concepts and solve problems. Factors affecting ELs' success in mathematics should also be taken into account. (See also the next section on Course Placement of English Learners.) There are a multitude of such factors that fall into at least one of seven characteristic types. These factors inform how educators can support ELs to achieve success in mathematics:

Van de Walle (2007) suggests specific strategies that teachers can incorporate into their mathematics instruction to support English learners:

- Let students know the purpose of the lesson and what they will accomplish during the lesson.
- Build background knowledge and link the lesson to what students already know.
- Encourage the use of each student's native language during group work while continuing to focus on English language development.
- Provide comprehensible input by simplifying sentence structure and limiting the use of non-essential vocabulary. Use visuals whenever possible.
- Explicitly teach vocabulary. Use a word wall and personal math dictionaries.
- Have students work in cooperative groups. This provides English learners with non-threatening opportunities to use language.

1. Limited prior or background knowledge and experience with formal schooling

- Some ELs may lack basic mathematics skills. EL students with limited prior schooling may not have the basic computation skills required to succeed in the first year of higher mathematics. ELs who enter U.S. schools in kindergarten benefit from participation in the same instructional activities as their non-EL peers, along with additional differentiated support based on student needs. Depending upon the level and extent of previous schooling they have received, ELs who enter U.S. schools for the first time in high school may need additional support to master certain linguistic and cognitive skills and fully engage in intellectually challenging academic tasks. Regardless of their schooling background or exposure to English, all ELs should have full access to the same high-quality, intellectually challenging, and content-rich instruction and instructional materials as their non-EL peers, along with appropriate levels of scaffolding.
- Some ELs may have prior or background knowledge, but it is important to avoid misconceptions of students' mathematics skill levels, especially when based upon their cultural background and upbringing.

2. Cultural differences

- Mathematics is often considered a universal language in which numbers connect people regardless of culture, religion, age, or gender (NYU Steinhardt 2009). However, mathematics learning styles vary by country and culture, and by individual students.
- The meanings of some symbols (such as commas and decimal points) and mathematical concepts differ according to culture and country of origin. This occurs frequently, especially when expressing currency values, measurement, temperature, and so on, and may impede an EL's understanding of the material being taught. Early on in the school year, teachers should survey their students to learn about the students' backgrounds and effectively address individual needs. It is important for teachers to inform themselves about particular aspects of their students' backgrounds, but also to see each student as an individual with distinct learning needs, regardless of cultural or linguistic influences.

3. Linguistics

Everyday language is very different from academic language, and when students struggle to understand and apply these differences, they may experience difficulties in acquiring academic language. Teachers should develop all of their students' understandings of how, why, and when to use different registers and dialects of English. Some of these challenges may include understanding mathematics-specific vocabulary that is difficult to decode, associating mathematics symbols with concepts, as well as the language used to express those concepts, and grasping the complex and challenging structure of the passive voice.

4. Polysemous words

Polysemous words have identical spellings and pronunciations, but different meanings that are based on context. For example, a *table* is a piece of furniture on which one can set food and dishes, but it is also a systematic arrangement of data or information. Similarly, an *operation* may be a medical procedure or a mathematical procedure; these meanings are different from each other in context, but they do have some relation to one another. The difference between *polysemes* and *homonyms* is subtle: polysemes have semantically related meanings, but homonyms do not.

5. Syntactic features of word problems

- The arrangement of words in a sentence plays a major role in understanding phrases, clauses, or the entire sentence. Complex syntax is especially difficult in the reading, understanding, and solving of word problems in mathematics (NYU Steinhardt 2009). Extra support should be given to ELs regarding syntactic features.
- Some algebraic expressions are troublesome for ELs, because if they attempt to translate the provided word order, the resulting equation may be inaccurate. For example: *A number x is 5 less than a number y .* It is logical to translate word for word when solving this problem, which would most likely result in the following translation: $x = 5 - y$. However, the correct equation would be $x = y - 5$.

6. Semantic features

As shown in the following table (adapted from NYU Steinhardt 2009), many ELs may find semantic features challenging.

Feature	Examples
Synonyms	<i>add, plus, combine, sum</i>
Homophones	<i>sum/some, whole/hole</i>
Difficult expressions	If . . . then; given that . . .
Prepositions	<i>divided into</i> versus <i>divided by</i> ; <i>above, over, from, near, to, until, toward, beside</i>
Comparative constructions	If Amy is taller than Peter, and Peter is taller than Scott, then Amy must be taller than Scott.
Passive structures	Five books were purchased by John.
Conditional clauses	Assuming x is true, then y . . .
Language function words	Words and phrases used to give instructions, to explain, to make requests, to disagree, and so on.

7. Text analysis

Word problems often pose challenges because they require students to read and comprehend the text, identify the question, create a numerical equation, and then solve that equation. Reading and understanding written content in a word problem are often difficult for native speakers of English as well as ELs.

When addressing the factors that affect ELs in instruction, it is essential for teachers to know the ELD proficiency-level descriptor that applies to each student in their classroom. The *emerging*, *expanding*, and *bridging* levels identify what a student knows and can do at a particular stage of English language development and can help teachers differentiate their instruction appropriately. The seven factors discussed above remain barriers for EL students if they are not addressed by teachers. Schools and districts are responsible for ensuring that all ELs have full access to an intellectually rich and comprehensive curriculum, via appropriately designed instruction, and that they make steady and accelerated progress in English language development, particularly in secondary grades.

Course Placement of English Learners

Educators must pay careful attention to placement and assessment practices for students who have studied mathematics in other countries and may be proficient in higher-level mathematics but lack proficiency with the English language. Indeed, a student's performance on mathematics assessments may be affected by his or her language proficiency. For example, in figure UA-3, results for students *A*, *B*, and *C* on the same test may look very similar even though the students' language and mathematical proficiency levels vary considerably. The design of the assessment needs to be mindful of this problem,